Interaction effects in topological insulators: New phases and phenomena

Ashvin Vishwanath
UC Berkeley

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1 Overview

In these lectures we will mainly be interested in how the concept of topological insulators generalizes when we include interactions. More generally, we discuss the interplay of symmetry and topology. Traditionally, phases of matter were distinguished on the basis of symmetry alone. On the other hand, fractional quantum Hall phases are examples of topological states whose essential character do not require a discussion of symmetry. However, topological insulators are an example of a new phase of matter that combines both symmetry and topology.

To generalize the concept of topological insulators to strongly interacting systems we will need some definitions to limit the set of states we study and hence make progress. Previously, the thinking was that phases like the integer quantum Hall state can occur in free fermion models, and the new physics that interactions bring are fractional Quantum Hall phases. Now we understand that there is some space between these two - there are states that retain the essential physical character of integer Quantum Hall states, but *require* interactions. Partly, the advances occurred by sharply defining what we mean by 'Integer Quantum Hall like' states - by identifying Short Range Entanglement as an essential property.

Throughout we will discuss phases with an energy gap in the bulk - and focus on zero temperature properties. Often we will be interested in new phases of matter, but some of the most striking results will expose phenomena connected to well known phases like topological insulators and superconductors that are obscured by the free particle description.

To whet your appetite, we begin my mentioning three striking theoretical results that emerge on including the effects of interactions. Establishing these will be the goal of these lectures.

- Integer quantum Hall states of electrons have long been believed to be characterized by, well, an integer ($\mathcal{Z}$)- which is the Hall conductance in units of e^2/h. We will see that this is modified in the presence of interactions - *actually the classification is by two integers* $\mathcal{Z} \times \mathcal{Z}$, and this family of states retains the essential properties of IQH phases.

- It was believed that if the conducting surface of a 3D topological insulator is made insulating, it must be because the time reversal symmetry is broken, either spontaneously or by application of external fields. It has recently been understood that you can have your cake and eat it too - that*there exist strongly interacting surface phases of a 3D topological insulator, that are insulating but retain time reversal symmetry.* The price you pay is that this state must have fractional excitations at the surface - i.e. ones with fractional charge and fractional (or anyonic) statistics. In fact, it must realize a particularly exotic version of fractional statistics - non-Abelian statistics. The simplest version of this state is closely related to the celebrated Read-Moore Pfaffian state, but with a twist.

- We are used to thinking of the surface states of topological phases (say of 3D phases) as being 'impossible' to realize in a purely 2D system with the same symmetries. Indeed this is generally true - even the new interacting surface phase of a topological insulator cannot be realized in a purely 2D system. However, the set of topological superconductors protected by time reversal symmetry (class DIII) are labeled by an integer ν according to the free fermion classification. Roughly, this counts the number of Majorana cones (which are like 'half' of a Dirac cone) present at the surface. From the free fermion point of view, all these surface states are 'impossible' 2D states. However, with interactions we will show that while $\nu = 1, 2, \ldots 15$ are all indeed impossible in 2D states, *the surface of* $\nu = 16$ *can be realized in a purely 2D but interacting model.* This also means that the integer classification is broken down $\mathcal{Z} \to \mathcal{Z}_{16}$.

All these are statements about adding interactions to many electron states. But to make progress we will need to take a diversion - and study topological phases of *bosons* or, equivalently, spins. Results there are integrally connected to a deeper understanding of interacting electronic topological insulators and superconductors. Moreover, they might be realized in experiments on ultra cold bosons or frustrated magnetic models. We will discuss some ideas along these lines, but it is fair to say, that conceptual theory is well ahead of experiments and model building in this area. However, the spectacular success of topological insulators in connecting with experiments makes us optimistic. There are even strongly correlated materials proposed to be in this phase. Perhaps some of you will contribute to this direction.

2 Lecture Outline

The provisional outline of lectures is as follows:

- Lecture 1 & 2:
  - Basic definitions and properties of short range entangled topological phases.
  - Examples of topological phases of bosons in 1D and 2D from condensing decorated defects.
  - Field theory of 2D SRE topological phases of bosons.
  - The integer quantum Hall effect revisited - revised classification of interacting Integer quantum Hall states.
  - Topological phase of bosons in 3D. Symmetry fractionalization and topological order. Surface topological order.
- Lecture 3: Interaction effects on topological insulators and superconductors. Surface topological order of electronic topological phases and their stability.